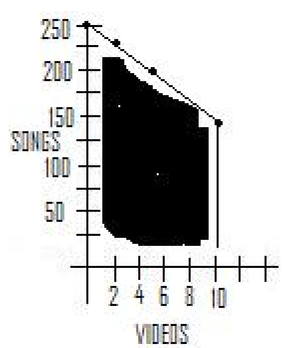
# $M^2$=Math Mediator Lesson 19: Linear Programming

**Total Recall (Warm-up) (15 minutes approx.)**

Total Recall: Exercise from yesterday's lesson on systems of inequalities.

For a 1 GB (Giga-Byte) music/video storage device, solve the following:

1. How many MB (Mega-Byte) are there in 1 GB (Giga-Byte)?
   a. Answer: 1000 (KB = $1000^1$ Bytes, MB = $1000^2$ Bytes, GB = $1000^3$ Bytes).
2. Songs take up approximately 4 MB of storage space and videos take up approximately 40 MB of space. What is the maximum number of songs and videos you can store on this 1 GB device?
   a. Answer: 250 songs/0 videos and 0 songs/25 videos. Ask the students to create an inequality for this relationship. The inequality, using variable s for songs and v for videos is: $4s + 40v \leq 1000$. Setting one variable to zero and solving gives the maximum amount of the other.
3. Add a constraint: Since videos take up a bunch of space, and most people listen to songs more than watching videos, limit the number of videos to 10 and graph the solution set of all the possible combinations of songs and videos, with the maximum videos being 10.
   a. Answer: With 10 videos at 40 MB each, the number of songs that would fit is: $4s + 40(10) \leq 1000$; $4s \leq 600$; $s \leq 150$




   b. This is the graph of the solution.
4. How would the graph change if the maximum number of videos was selected to be 8? Answer: the vertical line would move over to 8 and extend up to the sloped line.

**Direct Instruction (15 minutes approx.)**

**Linear Programming**

A teenager wanted to know how to plan out his/her time and make some money. They wanted to add paying activities to their schedule, while keeping some time set aside for fun and study time. They needed to set some goals, consider their options and plan out a feasible schedule.

They know that babysitting pays $10.00 per hour and yard work pays $15.00 per hour. They want to make $1000.00 in 6 months in order to be able to go on a trip with a group of their classmates. They also make a list of the activities they like and need to do during the week, including sleep, and estimate that they can comfortably set aside 8 hours for work, whether babysitting or yard work. They want to know if they can make the $1000.00 in 6 months working a maximum of 8 hours per week, and what are the constraints on babysitting jobs and yard work

©Copyright Math Mediator 2008

<table>
<tr>
<td></td>
<td>
or various combinations of each that will ensure they meet their goal.

From the information given, they can derive linear inequalities, called constraints or limits, which will give them possible solutions. Since they have broken down their work time to 8 hours per week, they convert 6 months to 24 weeks and dividing the $1000.00 by 24 gives them the goal of making $42.00 per week. The inequalities describing their goal, using b for babysitting hours and y for yard work hours, are:

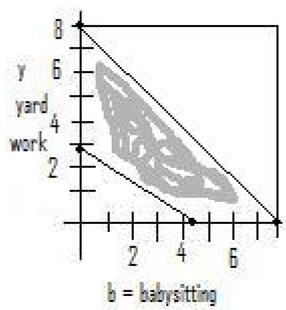
$10b + 15y \geq 42$ $10 per hour babysitting, $15 for yard work

$b + y \leq 8$ they limited their work time to 8 hours

$b \geq 0$ and $y \geq 0$ they can't have negative hours for any work

$b \leq 8$ and $y \leq 8$ neither of the times can be over their limit of 8 hours

One way to find if there are any solutions and what they are is to graph these linear inequalities:




This graph shows the linear constraints and the bounded area is the solution set, or feasible region. There are many combinations of working hours that will get them to their goal. Is 5 hours yard work and 4 hours of babysitting in one week a possible solution? No, because the total hours worked is over the 8 hour limit.
</td>
</tr>
<tr>
<td>Review (10 minutes approx.)</td>
<td>
U-DO: Somebody donated $160 to the person trying to raise the $1000.00 in the previous problem during the first day of their time frame. What are the new constraints and feasible region given this change?

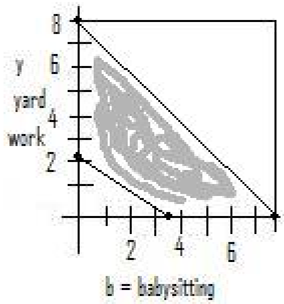
$10b + 15y \geq 35$ $840/24 is 35 per week they need to earn

$b + y \leq 8$ they limited their work time to 8 hours

$b \geq 0$ and $y \geq 0$ they can't have negative hours for any work

$b \leq 8$ and $y \leq 8$ neither of the times can be over their limit of 8 hours
</td>
</tr>
</table>

©Copyright Math Mediator 2008

| | |
|---|---|
| |   What has changed? The lower limit of hours per week. They can slack off during some weeks and make it up for others, as long as the average remains above the lower limit. |
| Activity (10 minutes approx.)<br>** Materials needed for this activity: string, cardboard, and a yardstick. | What exactly does linear mean? Any number of points that lie on the same line, right? We have analyzed 2 variable or 2 dimension linear inequalities, but what about lines in 3 dimensions, with 3 variables? Do they exist? Of course. How can we represent them? We will do a short exercise to show how lines are represented in 3 dimensions, or using 3 variables.<br><br>Step #1: Mark out on the floor or on a large piece of cardboard an x-y Cartesian plane. Label the axes and mark -5 through 5 on each.<br><br>Step #2: Using the yardstick as the positive z axis or plane, have students find the following points: (-1, 3, 4); (3, -2, 3); (2,2,2); and (2, 2, 0)<br><br>Step #3: Using the string, have two students put their end of the string on the following points, since a line is defined by two points: (0,0,0) and (2,2,0). How many dimensions does this line lie in? (Three actually, but z is zero). Second set of points for a line: (1,3,3) and (4,5,1). This line has 3 dimensions to it. How would you define the slope of this 3-D line? The slope would have to be defined per axis pair (i.e. slope in x-y plane, slope in x-z plane, and slope in y-z plane).<br><br>Step #4: Using a longer string have 3 students show how three points defines a plane: (0, 0, 0); (4, 0, 0); and (0, 2, 0). In what dimensions does this plane lie? The x-y plane. Second set of points to define a plane: (0, 0, 0); (3, 0, 3); and (0, 4, 0). Show how the plane is now described by the surface of the triangle formed by the string. |
| Wrap-up and homework assignment (5 minutes approx.) | Equations with 3 variables describe all the points that are on a plane, just like equations with 2 variables describe all the points on a line. Inequalities with 3 variables describe points above or below a plane. Planar Programming, just like Linear Programming, uses planes as boundaries or constraints to define a feasible region.<br><br>Tomorrow we will look at equations with three variables: $x + y + z = 5$.<br><br>Wrap up closing comments and housekeeping. |

©Copyright Math Mediator 2008