# M$^2$=Math Mediator    Lesson 20: Systems of Equations with Three Variables

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| Total Recall (Warm-up) (5 minutes approx.) | Total Recall: Exercise from yesterday's lesson on Linear Programming.<br>1. Is the equation x = 4 a point? A line? Or a Plane? Answer: It can be any of these, depending on how you define the solution set. On a number line, it would be a point. On the x-y plane it would be a line. On a 3 dimensional graph, it would be a plane.<br>2. If you are given 2 points (1, 2, 3) and (0, 4, 0); is this enough information to define a line? A plane? Answer: Only a line.<br>3. How many points do you need to define a plane? Answer: 3.<br>4. What does the equation 3x + 2y = 5 define? Answer: a line<br>5. What does the equation 2x – 3y – 4z = 16 define? Answer: a plane |
| Direct Instruction (20 minutes approx.)<br><br>CA. STD 2.0 | By now the students should know how to solve systems of equations involving 2 variables and two equations. Today the objective is to introduce systems of three equations and 3 variables.<br><br>The application is yearbook economics. The yearbook committee discovers that they can get 300 books published at 12 cents per page and $12.00 per cover. On top of meeting the costs, they want to raise $2000.00 for school clubs. They will allow advertisers and can earn $3.00 per advertising page. However, they would like to restrict the advertising pages to a ratio of 10 to 1. 20 being the number of photo and school related pages, to 1 page of advertising. They will charge people 20% over their cost to help out school clubs. Using p for number of school pages, a for advertising pages and c for book cost, the equations from this given information are:<br><br>1) 1.2(12 + 0.12(p + a)) = c    The cost of one yearbook, with 20% markup.<br>2) 300(3a – c) = 2000   300 books times net advertising minus cost is $2000<br>3) 10a = p    The ratio of advertising pages to school related pages.<br><br>Three equations with 3 variables. 3 goals to be met, want one solution.<br><br>With systems of two equations, there were two methods of solving. One was elimination, the other was substitution. We use the same principle with systems of three equations.<br><br>**Step #1**: Eliminate one variable by elimination or substitution from any two of the three equations.<br><br>**Step #2**: Select the remaining equation and one of the two you previously used and eliminate the same variable you eliminated in step #1.<br><br>**Step #3**: Take the resulting equations from Step #1 and Step #2, they should have only two of the same variables in each, or less, and then eliminate one variable from them by elimination or substitution. This gives you the solution for one variable.<br><br>**Step #4**: Substitute the solution from Step #3 into one of the two variable |

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| | equations from Step #1 or 2 and solve for the second variable.<br><br>**Step #5**: Take those two solutions from Step #4 and Step #3 and substitute them into one of the original equations having all three variables and solve for the third variable.<br><br>**Step #6**: Check your solution by substituting the solutions into all initial equations.<br><br>Now, lets go back through these steps using the yearbook equations.<br><br>**Step #1**: Equation 2) 300(3a – c) = 2000 and Equation 3) 10a = p<br>Equation 3 can also be represented as a = p/10, so we can substitute that into equation 2 and we have a new equation: 300(3(p/10) – c) = 2000 or 90p – 300c = 2000<br><br>**Step #2**: Equation 1) 1.2(12 + 0.12(p + a)) = c and Equation 3) 10a = p<br>Substituting a = p/10 into Equation 1 gives: 1.2(12 + 0.12(p + (p/10))) = c or 1.2(12 + 0.12(1p + 0.1p)) = c or 1.2(12 + 0.12(1.1p)) = c or 1.2(12 + 0.132p) = c or 14.4 + .1584p = c<br><br>**Step #3**: The two equations that we have are: 90p – 300c = 2000 and 14.4 + 0.1584p = c. Substituting the value of c in the second equation into the first equation gives us: 90p – 300(14.4 + 0.1584p) = 2000; reducing this by first distributing the 300 gives: 90p – 4320 – 47.52p = 2000 and then combining like terms: 42.48p = 6320 and p = 149 pages rounding up.<br><br>**Step #4:** Substitute p = 149 into Step #2 equation: 14.4 + .1548p = c gives us the result of 14.4 + .1548(149) = c and c = 37.46 dollars per yearbook<br><br>**Step #5:** Substitute p = 149 and c = 37.46 into the original equation #1 which was: 1.2(12 + 0.12(p + a)) = c is: 1.2(12 + 0.12(149 + a)) = 37.46. Then we solve for a, first by dividing both sides by 1.2 to get: 12 + 0.12(149 + a)) = 37.46/1.2 = 31.221. Then we subtract both sides by 12 to get 0.12(149 + a) = 19.221. Then divide both sides by 0.12 to obtain: 149 + a = 160.175 and then at last we subtract both sides by 149 to get a = 11.175 or 12 pages.<br><br>The solution is that the yearbook will have 149 pages of photos and school memorabilia, 12 pages of advertizing, and will cost 37.46 dollars to make it and will sell at a 20% markup or 45 dollars.<br><br>In a real situation, there would be inequalities in the place of equal signs, because you would actually want to make at least $2000 for the profit going to school clubs. |
| Review and practice (15 minutes approx.) | U-DO: Students try this system of three equations:<br><br>A hotel manager wants to hire maids, front desk people and technical/maintenance people with a budget of $200,000.00 per year. The front desk people get paid 1.5 times the maids, technical people get 1.25 times the front desk people. He needs 3 maids, 3 front desk people and 2 techs. How much should he offer them for the position? |

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| | Solution is to set three variables and obtain 3 equations. Let m = maid's pay, f = front desk people's pay, and t = tech's pay.<br>Equation #1: 3m + 3f + 2t = 200,000 he has $200k to split between all<br>Equation #2: 1.5m = f front desk people get paid 1.5 times maids<br>Equation #3: 1.25f = t techs get paid 1.25 times maids<br>ANSWER:<br>1,2: substitute 1.5m for f in eq. 1 to get 3m +3(1.5m) + 2t = 200,000<br>2,3: substitute 1.5m for f in eq. 3 to get: 1.25(1.5m) = t<br>Then take the 2,3 solution of t = 1.25(1.5m) = 1.875m into the 1,2 solution to get: 3m + 4.5m + 2(1.875m) = 200,000 and combining all m's: m = 17,800. Take that an put it into the 2,3 solution to get t = 1.25(1.5m) = 33,300. Then to get f, use the original equation #2: 1.5(m) = f = 1.5(17,800) = 26,700. |
| Practice (5 minutes approx.) | Solve for these three simultaneous or systems of equations:<br>1. 10x + 3y + 2z = 99<br>2. 4x + 8y + 2z = 78<br>3. 2x + 3y + z = 33.6 Answer: x = 8; y = 5.4; z = 1.4 |
| Direct Instruction; practice and assessment: (8 minutes approx.) ** Cardboard pieces here help to show three planes intersecting. | There are three possible solutions for systems of three equations in three variables.<br>1. The solution is one point: as in all our examples today.<br>2. The solution is infinite: it is a line where the planes intersect or it is three equations describing the same plane.<br>a. 1x – 2y – z = 8; 2x – y – z = 0; and 3x – 6y + 3z = 24; when you try to eliminate a variable from 1 and 3, you end up with 0 = 0; which is true, so that means the solution is a line, infinite points.<br>3. The solution is an empty set: two or more planes are parallel.<br>a. 2x – 4y – z = 10; 4x – 8y – 2z = 16; and 3x + y + z = 12; when you try to eliminate a variable from 1 and 2, you end up with 0 = 32 or 0 = 18, neither is true, which means there is no solution. |
| Wrap-up (2 minutes approx.) | Wrap up closing comments and housekeeping. |